

Graph theory, learning process and student performance

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Overview

We provide a framework for studying the dynamics involved in student performance.

The framework takes into account, among other things, peer relationships and student categories at any given time.

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Question: Why study graph theory?

It is used, among other things, in:

1 Communication networks -

- internet,
- GPS - finding shortest paths,
- resource location.

2 Health sciences -

- predicting outbreak of diseases,
- Understanding of viral propagation behaviour,
- drug manufacturing.

3 Sociology,

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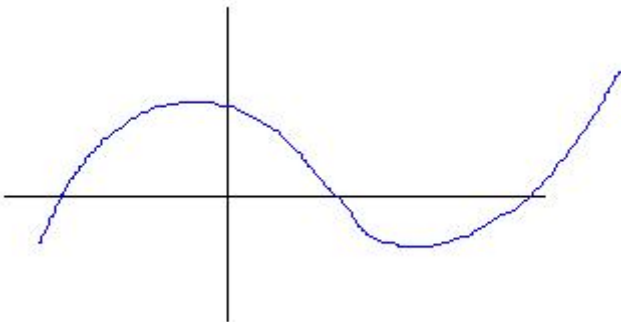
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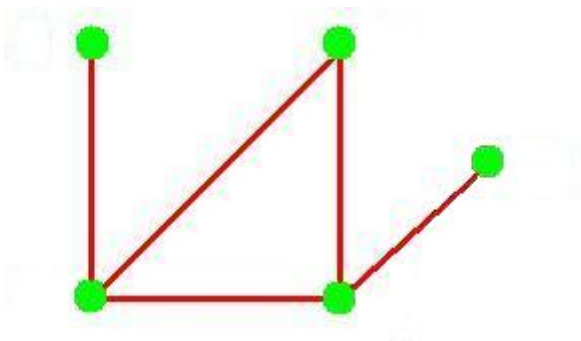
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Graphs

Rem: A graph is not ...



but 'a collection of **points (vertices)** and **lines (edges)**':



Rem: Graphs provide a natural framework for explaining almost every process where some **objects** are **linked** for the purposes of communication or exchange of **that-which-flows** in the network, e.g.,

communication networks

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- **terminals** = vertices;
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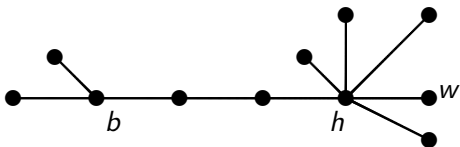
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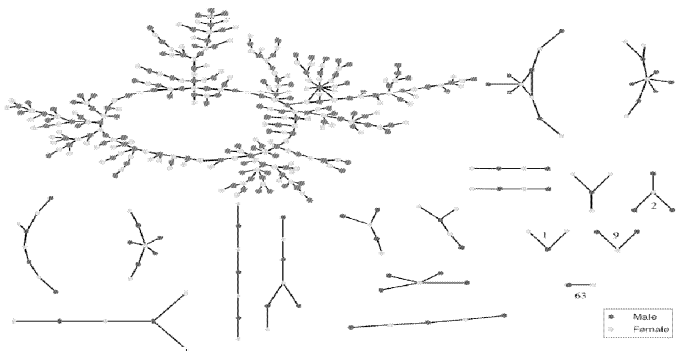
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Sexual Network at Jefferson High



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We model the class by a graph where:

- **students** = vertices;
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At any given time, each student is in one of the three categories:

- **Weak**;
- **Average**;
- **Performing**.

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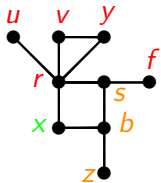
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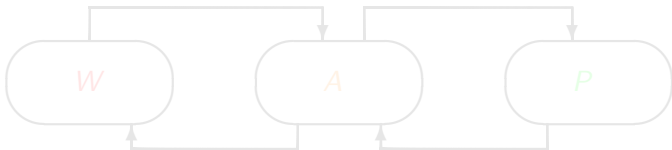
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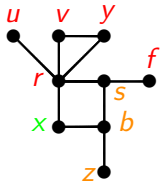
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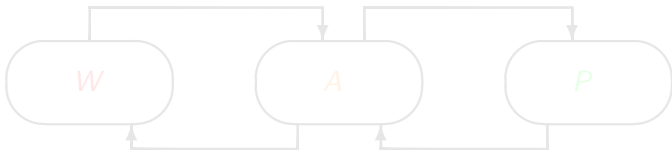
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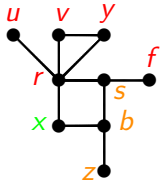
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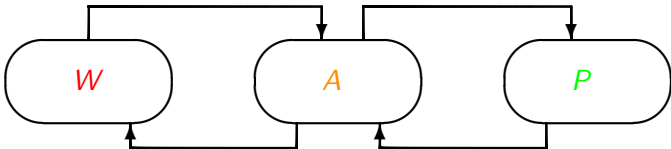
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Progression rules: weak student ...

Rw: A **weak** student becomes **average** in the next time step if the density of performing students among his/her friends exceeds a fraction α^* ; otherwise the student stays weak.

This is how we incorporate the fact that **academic success is contagious** [(Hanushek et al, 2003), (Blansky et al, 2013)].

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Progression rules: performing student ...

R_p: A **performing** student becomes **average** in the next time step if the density of weak students among his/her set of friends exceeds a fraction γ^* ; otherwise the individual remains in the performing category.

- This is how we capture the decrease of performance due to the negative influence of weak students as peers [(Lavy et al, 2011)].
- This follows a majority process where humans follow the crowd in doing things.

We omit here a similar rule, R_a, for progression of average students.

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Results

Our results, derived from **rigorous mathematical proof**, follow:

Let V be the class. Denote the category of student v at time t by $c_t(v)$.

Theorem 1: There exists a set $W' \subseteq V$ such that

$$c_t(W') = w \text{ for all } t \geq 0.$$

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Some remarks ...

- 1 Theorem 1 identifies the most vulnerable weak students W' who, in the absence of intervention, will remain weak during the entire duration of study. This identification is complicated and not straightforward!

Recommendations:

Assume delivery methods early on that aim at reducing the size of W' .

- If you have enough resources, target W' .
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More results ...

- 1 We also found that from time to time, during the study period, **strong students emerge**, having the ability to:
 - act as local hubs of knowledge,
 - sustain good performance within their neighbourhoods.

These strong students can be targets for SI and postgraduate studies.

- 2 Finally, our model has shown that **pass rates alone**, or any other parameter based on student scores, cannot adequately give a **full picture of the potential of students to perform**. These should be considered in conjunction with the **student network structure**. For instance, student A and student B may both be **12% level students**, but the **position of student A in the network** may give student A better chances of performing than student B.

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