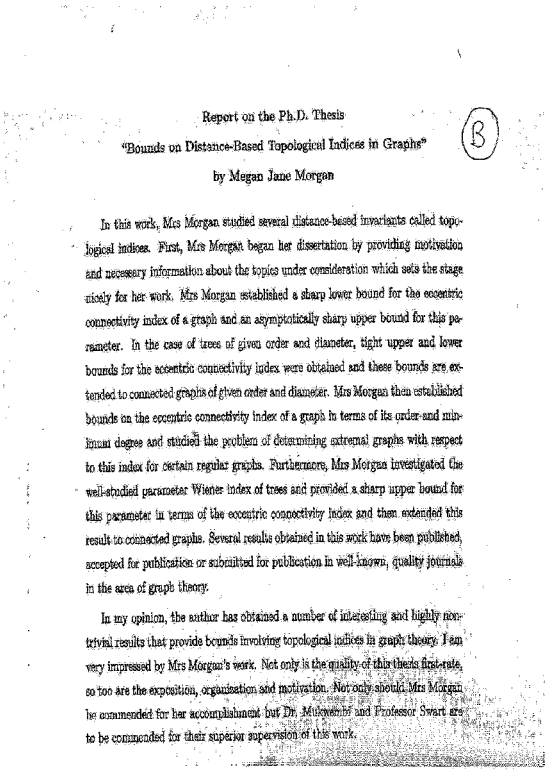
7.12 Annexure 2I: External examiners' reports



Report on the Ph.D. Thesis

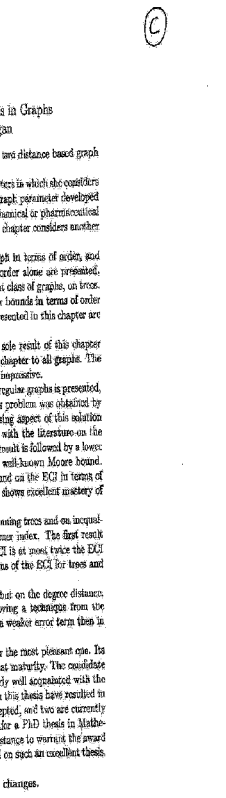
"Bounds on Distance-Based Topological Indices in Graphs"

by Megan Jane Morgan

(B)

In this work, Mrs Morgan studied several distance-based invariants called topological indices. First, Mrs Morgan began her dissertation by providing motivation and necessary information about the topics under consideration which sets the stage nicely for her work. Mrs Morgan established a sharp lower bound for the eccentric connectivity index of a graph and an asymptotically sharp upper bound for this parameter. In the case of trees of given order and diameter, tight upper and lower bounds for the eccentric connectivity index were obtained and these bounds are extended to connected graphs of given order and diameter. Mrs Morgan then established bounds on the eccentric connectivity index of a graph in terms of its order and minimum degree and studied the problem of determining extremal graphs with respect to this index for certain regular graphs. Furthermore, Mrs Morgan investigated the well-studied parameter Wiener index of trees and provided a sharp upper bound for this parameter in terms of the eccentric connectivity index and then extended this result to connected graphs. Several results obtained in this work have been published, accepted for publication or submitted for publication in well-known, quality journals in the area of graph theory.

In my opinion, the author has obtained a number of interesting and highly non-trivial results that provide bounds involving topological indices in graph theory. I am very impressed by Mrs Morgan's work. Not only is the quality of this thesis first-rate, so too are the exposition, organisation and motivation. Not only should Mrs Morgan be commended for her accomplishment but Dr. Mukwembi and Professor Swart are to be commended for their superior supervision of this work.



(C)

Examiner's Report on

Bounds on Distance Based Topological Indices in Graphs

PhD Thesis submitted by M.J. Morgan

In this thesis, the candidate presents a collection of results on two distance based graph parameters, also called topological indices.

After an introductory chapter, the candidate presents four chapters in which she considers in detail the eccentric connectivity index (ECI), a relatively new graph parameter developed with the aim of finding an indicator that would allow to predict chemical or pharmaceutical properties of molecules from their topological structure. The final chapter considers another topological index, the degree distance.

In Chapter 2, a lower bound on the ECI of a connected graph in terms of order, and upper bounds in terms of order and diameter, and in terms of order alone are presented. The second part of the chapter considers the ECI on an important class of graphs, on trees. Upper bounds on the ECI in terms of order, and upper and lower bounds in terms of order and diameter are presented. It is worth noting that all bounds presented in this chapter are sharp.

Chapter 3 returns to the ECI of all connected graphs. The sole result of this chapter extends a lower bound on the ECI of trees given in the previous chapter to all graphs. The proof of this result has a considerable degree of complexity, it is impressive.

In Chapter 4, an essentially sharp upper bound on the ECI of regular graphs is presented, thus solving an open problem by Dobrić et al.The solution of this problem was obtained by applying results on the average eccentricity. A particularly pleasing aspect of this solution is that it shows that Mrs Morgan familiarised herself not only with the literature on the ECI, but also with results in related areas of graph theory. This result is followed by a lower bound on the ECI of regular graphs which is derived from the well-known Moore bound. The final part of the chapter consists of an essentially sharp bound on the ECI in terms of order and minimum degree. Its proof is highly nontrivial, and it shows excellent mastery of the techniques used in this part of graph theory.

Chapter 5, the shortest chapter of this thesis, focusses on spanning trees and on inequalities relating the ECI and the oldest topological index, the Wiener index. The first result shows that every connected graph has a spanning tree whose ECI is at most twice the ECI of the original graph. Then bounds on the Wiener index in terms of the ECI for trees and for connected graphs are presented.

Chapter 6, the last chapter of the thesis, is not on the ECI but on the degree distance. It fully settles a 13 year old conjecture by Tomescu by improving a technique from the literature, which almost solved Tomescu's conjecture, but with a weaker error term than in the conjecture.

Among the PhD theses I have examined, this has been by far the most pleasant one. Its presentation - in motivation, statements and proofs - shows great maturity. The candidate has clearly taken great care in preparing this thesis. She is clearly well acquainted with the typical methods of this field of research. The results obtained in this thesis have resulted in five publications, of which two have appeared, one has been accepted, and two are currently under review. Five publications in total is well above average for a PhD thesis in Mathematics. The results obtained are without doubt of sufficient substance to warrant the award of a PhD for this thesis. The candidate should be congratulated on such an excellent thesis.

I recommend acceptance of the thesis as is, with no changes.