7.14 Annexure 3B: Tutorial test and marking guide

Math 105 Tutorial Test 5

1. Find the derivative of the function
$$f(x) = \ln(x^3 + x - 1).$$
(2)

2. Let $f(x) = xe^x$. Find $f'(0)$. (2)

3. Identify relative maxima and relative minima, if any, of the function
$$f(x) = x^2 - x + 1.$$
(3)

4. Find the intervals where the function
$$f(x) = \frac{1}{x}$$
is increasing and the intervals where it is decreasing. (3)

MATH105: Tutorial test 5 solutions

1. Since $\frac{d}{dx}\left(\ln g(x)\right) = \frac{g'(x)}{g(x)}$ ✓(½) idea, we have
$$f'(x) = \frac{3x^2+1 \text{ ✓(1)}}{x^3+x-1 \text{ ✓(½)}}$$

2. Note that $f'(x) = e^x + x.e^x$. ✓(1)
$\therefore$ $f'(0) = e^0 + 0.e^0 = 1$. ✓(1)

3. Note that $f'(x) = 2x - 1 = 0$ ✓(½) iff $x = \frac{1}{2}$. ✓(½)
Now $f''(x) = 2$ ✓(½), & so $f''(\frac{1}{2}) = 2 > 0$ ✓(½) $\therefore$
f has a relative minima ✓(½) at $x = \frac{1}{2}$. ✓(½)
(Please note that students may use the first derivative test.)

4. Differentiating: $f'(x) = -\frac{1}{x^2}$ ✓(½) $\neq 0$ for all x,
& it is undefined if $x = 0$. ✓(½)
The open intervals are therefore,
$(-\infty, 0)$ & $(0, \infty)$ ✓(½). Now
$f'(-1) = -1 < 0$; hence f is decreasing ✓(½)
✓(½) in $(-\infty, 0)$.
Also $f'(1) = -1 < 0$; hence again f is decreasing in $(0, \infty)$. ✓(½)